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Few-Body Systems

Relativistic Hamiltonian Dynamics and Few-Nucleon Systems

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Abstract. We present a preliminary calculation of the electromagnetic form factors of ^{3}He and ^{3}H, performed within the Light-Front Hamiltonian Dynamics. Relativistic effects show their relevance even at the static limit, increasing at higher values of momentum transfer, as expected.

1 Introduction

The *standard model* of Few-Nucleon Systems, where Nucleon and pion degrees of freedom are taken into account, is already at a very sophisticated stage, and many efforts are presently carried on to retain all the *general principles* of a theory with a fixed number of constituents. In particular, to satisfy Poincaré covariance appears a quite reachable goal and at the same time a very compelling requirement in view of the forthcoming measurements of electromagnetic (em) form factors (ff's) for A=3,4, in the region of few GeV's [1]. In order to extract unambiguous signatures of effects beyond the *standard model* of Few-Nucleon Systems one should develop a fully field-theoretical approach, based on the Bethe-Salpeter equation, but the difficulties are well-known, and therefore one has to consider alternative approaches, like 3-D reductions (see, e.g., [2] for A=2 and [3] for the work in progress for A=3) or the Relativistic Hamiltonian Dynamics (RHD) framework, suggested by Dirac in a seminal paper [4].

Our aim is to construct, within the Light-Front form of RHD, a relativistic approach for Few-Nucleon System that i) retains the whole *successful phenomenology* already developed and ii) includes, *in a non perturbative way*, the relativistic features requested by Poincaré covariance. In order to have a strong and immediate comparison with the experiments we have focused our efforts on the development of Poincaré covariant calculations of the electromagnetic ff's, extending our approach from the Deuteron [5] to the Trinucleon. The adopted Bakamjian-Thomas (BT) procedure (see, e.g., [6]) allows us to exploit realistic wave functions for Few-Nucleon Systems (for A=3, see, e.g., [7]) in order to evaluate matrix elements of a Poincaré covariant current operator [8] for an interacting system. The relativistic effects imposed by Poincaré covariance materialize in the relativistic kinematics and in the presence of the so-called Melosh rotations (see, e.g., [6]), that allows one to use the standard Clebsh-Gordan machinery to obtain many-nucleon wave functions with the correct angular coupling.

2 Formalism

As well-known [6], Light-Front (LF) RHD has some appealing features, like the largest number of kinematical Poincaré generators (given the symmetry of the *initial* hypersurface $x^+ = 0$) and the simplest procedure for separating out the center of mass motion from the intrinsic one, in strict analogy with the non-relativistic procedure. Moreover, it shares with the other two RHD's (Instant and Point forms) the rigorous fulfillment of the Poincaré covariance, for a system with a fixed number of constituents. In some sense, RHD's fall between non-relativistic quantum mechanics and the local, relativistic field theory.

For an interacting system, an em current operator, J^μ, that fulfills the extended Poincaré covariance (i.e. including parity and time reversal) and Hermiticity, can be constructed by a suitable auxiliary operator, j^μ, that fulfills rotational covariance around the z-axis in a Breit frame ($\mathbf{P}_f + \mathbf{P}_i = 0$), where the $\perp$ component of the momentum transfer is vanishing ($\mathbf{q}_\perp = 0$) [8]. Note that such a frame is *different* from the Drell-Yan one, where $q^+ = 0$. In general [8], the matrix elements $\langle P_f | J^\mu | P_i \rangle$, still acting on internal variables, are directly given by the matrix elements of the auxiliary operator j^μ, evaluated in the chosen Breit frame. A minimal Ansatz for a *many-body* auxiliary operator is built from i) the free current (a one-body operator) and ii) the $\perp$ component of the angular momentum operator $\boldsymbol{S}$ (a many-body operator in LF) as follows

$$j^\mu_{fi}(q\hat{e}_z) = \frac{1}{2}\left[\mathcal{J}^\mu_{fi}(q\hat{e}_z) + L^\mu_\nu[r_x(-\pi)]\ e^{\mathrm{i}\pi S_x}\ \mathcal{J}^\nu_{if}(q\hat{e}_z)^*\ e^{-\mathrm{i}\pi S_x}\right] \qquad (1)$$

with $\mathcal{J}^\mu_{fi}(q\hat{e}_z) = \Pi_f\ J^\mu_{free}(0)\ \Pi_i$, $\Pi \equiv$ projector onto the states of the (initial or final) system and $\boldsymbol{S} \equiv$ the LF-spin operator of the system as whole: it acts on the "internal" space and is unitarily related to the standard angular momentum operator through the Melosh operators. Let us remind that $J^\mu_{free}(0)$ is the proper sum over A=2,3 free Nucleon current given by $J^\mu_N = -F_2[(p'^\mu - p^\mu)^2](p^\mu + p'^\mu)/2M + \gamma^\mu(F_1[(p'^\mu - p^\mu)^2] + F_2[(p'^\mu - p^\mu)^2])$, with $F_{1(2)}[(p'^\mu - p^\mu)^2]$ the Dirac (Pauli) Nucleon ff. In our Breit frame, charge normalization and current conservation (for $M_f = M_i$) can be fulfilled by imposing $\mathcal{J}^-(q\hat{e}_z) = \mathcal{J}^+(q\hat{e}_z)$[8, 5].

For evaluating matrix elements of $j^\mu(x)$, eigenstates of the interacting system are needed. To this end one can use the "non relativistic solutions", but with Melosh Rotations in the angular part, if the interaction $V \equiv M_{int} - M_0$ (where $M_{int}(M_0)$ is the mass operator of the interacting (free) system) can be embedded in a BT framework. The BT construction for obtaining interacting Poincaré generators suggests a necessary (not sufficient) condition [6] on the interaction: V must depend upon intrinsic variables combined in scalar products, i.e. $[\boldsymbol{\mathcal{B}}_{LF}, V] =$

Table 1. Magnetic moment (in nuclear magnetons) and quadrupole moment (in fm^2) for the Deuteron [5]; P_D is the D-state percentage. The corresponding exp. values are: $\mu_{exp} = 0.857406(1)$ and $Q_{exp} = 0.2859(3)$

Interaction	P_D	μ_D^{NR}	μ_D^{LFD}	Q_D^{NR}	Q_D^{LFD}
CD-Bonn	4.83	0.8523	0.8670	0.2696	0.2729
Nijm1	5.66	0.8475	0.8622	0.2719	0.2758
RSC93	5.70	0.8473	0.8637	0.2703	0.2750
Av18	5.76	0.8470	0.8635	0.2696	0.2744

$[\boldsymbol{S}_0, V] = [P_\perp, V] = [P^+, V] = 0$, where $\boldsymbol{\mathcal{B}}_{LF}$ are the LF boosts, $\boldsymbol{S}_0 \equiv$ the angular momentum operator for the non interacting case (since $S_0^2 = S_{int}^2$ and $S_{0,z} = S_{int,z}$, the eigenvalues of S_0^2 and $S_{0,z}$ can label the eigenstates of the interacting system). The non relativistic interaction fulfills the above requirements.

3 EM observables for A=2 and A=3 nuclei

First, let us briefly review our results for the Deuteron, and then we present preliminary calculations for the trinucleon case.

Magnetic and quadrupole moments of the Deuteron (see Table 1), as well as the ff's, $A(Q^2)$ and $B(Q^2)$ (see Fig. 1), and the tensor polarization $T_{20}(Q^2)$ have been calculated [5], showing the great relevance of the Poincaré covariance, even at low Q^2. From our results, one could argue that the role of MEC and pair contributions should shrink, but clearly a direct evaluation of two-body contributions is mandatory for a closer comparison with experiments.

A very interesting topic, related indeed to both A=2 and A=3 nuclei, given the absence of free neutron targets, is the extraction of the ratio of Nucleon structure functions, $r(x) = F_2^n(x)/F_2^p(x)$, in the Bjorken limit. This extraction has been analyzed adopting a standard approach [10], and here we would simply recall that a LF approach could give remarkably different results. By using the Impulse Approximation, the Deuteron structure function can be expressed through a convolution of the Nucleon structure functions and the distribution probability, $f^D(z)$, to find inside ^{2}H a nucleon with LF momentum z. In the usual approach, $f^D(z)$ is basically an adapted Instant-form approach with an off-mass-shell struck Nucleon, while within a rigorous LF approach [11] it becomes

$$f_{LF}^D(z) = \int d\boldsymbol{p}\; n^D(|\boldsymbol{p}|)\; \delta\left(z - \xi \frac{M_D}{M}\right) \tag{2}$$

where $n^D(|\boldsymbol{p}|)$ is the Nucleon momentum distribution in the Deuteron, $M(M_D)$ the Nucleon (Deuteron) mass, $\xi = p^+/P^+ = (\sqrt{M^2 + |\boldsymbol{p}|^2} + p_z)/2\sqrt{M^2 + |\boldsymbol{p}|^2}$.

Following [10] one can extract the ratio $r(x)$ from the experimental data for the Deuteron structure function and a suitable recurrence relation, where $f_D(x)$ has to be considered. In Fig. 2, one sees the interesting effect on the Deuteron structure function and on $r(x)$ (cf. $x \to 1$), produced by a LF $f^D(z)$, Eq. (2).

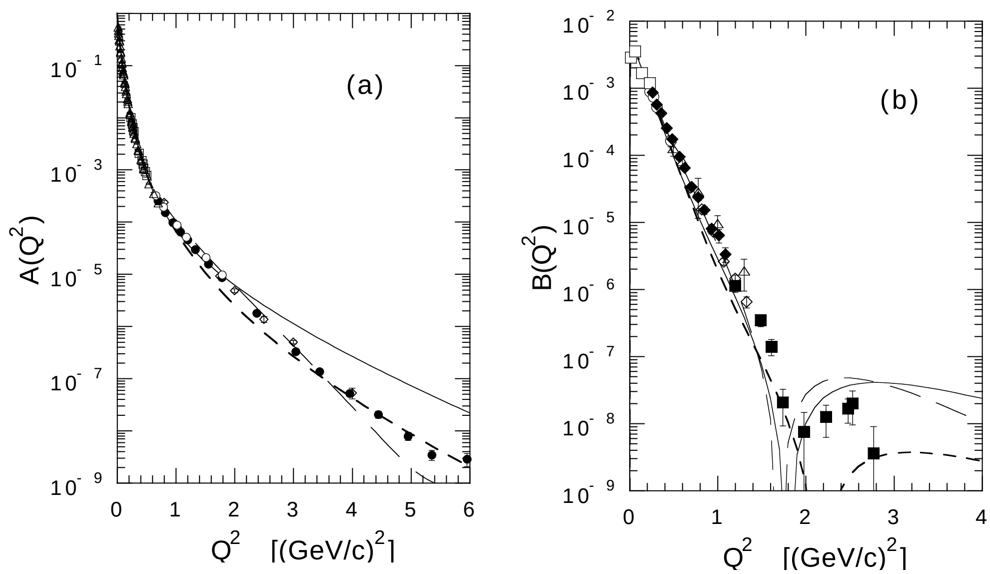

Figure 1. *RSC* $N-N$ interaction + Gari-Krümpelmann Nucleon ff's. [9]. Solid line: LF full result with the Poincaré covariant current operator, in the Breit frame where $\mathbf{q}_\perp = 0$. Dashed line: the same as the solid line, but the argument of the Nucleon ff's, $(p_1' - p_1)^2$, is replaced by $-Q^2$. Long-dashed line: non relativistic result in the same Breit frame.(After Ref. [5])

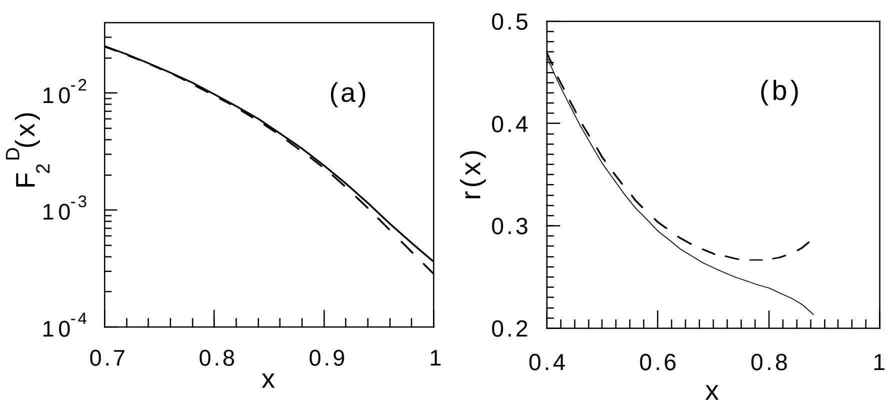

Figure 2. Left panel: the ^{2}H structure function, $F_2^D(x)$, vs. the Bjorken variable x. Solid lines: LF calculations, see Eq. (2). Dashed lines: standard approach calculations. Right panel: the same as in the left panel but for the ratio $r(x) = F_2^n(x)/F_2^p(x)$. AV18 NN interaction and the model of Ref. [12] for the Nucleon structure functions are adopted. (After Ref. [11]).

Trinucleon em observables, see Table 2 and Figs. 3-4, can be evaluated analogously to ^{2}H case, but with a very cumbersome angular coupling. In this preliminary calculation (S, P and D waves only for observables in Table 2) the A=3 wave function [7] without Coulomb interaction and corresponding to AV18 [14] NN interaction has been used. The charge and magnetic ff's are calculated from the matrix elements of a Poincaré covariant current as follows

$$F_{ch}^{T_z}(Q^2) = \frac{1}{2}\, Tr[\mathcal{I}^+(T_z)] \qquad\qquad F_{mag}^{T_z}(Q^2) = -i\frac{M}{Q}\, Tr[\sigma_y\, \mathcal{I}_x(T_z)]$$

Table 2. Trinucleon magnetic moments and charge radii. Preliminary calculation with S+P+D waves: $\mathcal{P}_{S+S'}(Av18) \sim$ 91.4% $\mathcal{P}_P(Av18) \sim$ 0.07% $\mathcal{P}_D(Av18) \sim$ 8.5%.

Theory	$\mu(^3\mathrm{He})$	$\mu(^3\mathrm{H})$	$r_{ch}(^3\mathrm{He})$	$r_{ch}(^3\mathrm{H})$
NR(S)	-1.723(2)	2.515(2)	1.841(3) fm	1.798(3) fm
LFD(S)	-1.7860(2)	2.6034(2)	1.867(3) fm	1.821(3) fm
NR(S+S')	-1.7093(2)	2.515(2)	1.896(3)	1.726(3) fm
LFD(S+S')	-1.768(2)	2.600(2)	1.919(3)	1.772(3) fm
NR(S+S'+P+D)	-1.769(2)	2.579(2)	1.882(4) fm	1.714(4) fm
LFD(S+S'+P+D)	-1.839(2)	2.674(2)	1.906(4) fm	1.754(4) fm
Exp.	-2.1276	2.9789	1.959(30)fm	1.755(86) fm

where $\mathcal{I}^r_{\sigma'\sigma}(T_z) \equiv \langle \Psi^{\frac{1}{2}T_z}_{\frac{1}{2}\sigma'}, P_f | \; \mathcal{J}^r \; | \Psi^{\frac{1}{2}T_z}_{\frac{1}{2}\sigma}, P_i\rangle$ with $r = +, 1$ (cf Eq. (1)).

4 Conclusions & Perspectives

In order to construct a *standard model* for Few-Nucleon Systems it is necessary to consider relativistic effects. For a fixed number of constituents, one can be a Poincaré covariant approach by adopting a Light-Front RHD and the Bakamjian-Thomas procedure. Within such an approach and the em current operator suggested in Ref. [8], the em observables of the A=3 nuclei have been calculated for the first time (S, P and D waves for the observables in Table 2 and S-wave only for ff's in Figs. 3-4). The few % effect for em observables at $Q^2 = 0$, in the correct direction, is very encouraging. Moreover, the sizable effect at high Q^2 indicates the essential role played by relativity for analyzing the em ff's in the region of few GeV's. A full calculation, with a systematic analysis of the pair contribution (Z-diagram) and current operators fulfilling the Ward-Takahashi Identity, will be presented elsewhere.

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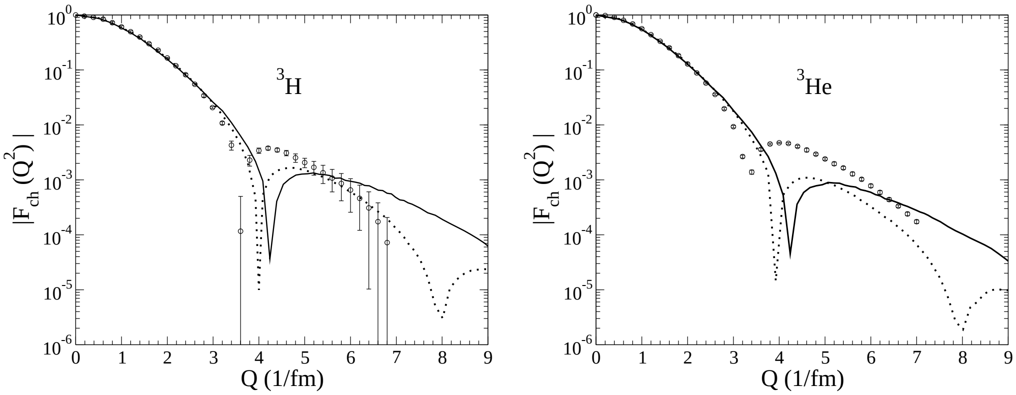

Figure 3. Trinucleon charge ff vs Q, only S-wave. Left panel: ^{3}H. Right panel: ^{3}He. Solid line: LFD calculations in a Breit frame where $\mathbf{q}_\perp = 0$. Dotted line: non relativistic calculations in the same frame. AV18 trinucleon wave function [7] and Gari-Krümpelmann Nucleon ff's [9] have been adopted. Data from [13].

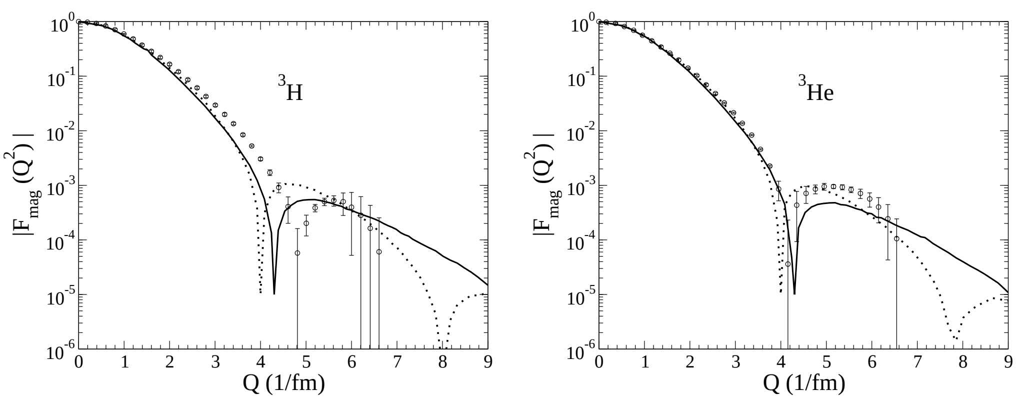

Figure 4. The same as in Fig. 3 but for the trinucleon magnetic ff.

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